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Nonlinear Processes in Geophysics (2000) 7: 49–58

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Received: 10 May 1996 – Accepted: 14 April 1999

Abstract.
The work is concerned with the results of theoretical and laboratory modelling the processes of the large-scale structure generation under turbulent convection in the rotating-plane horizontal layer of an incompressible fluid with unstable stratification. The theoretical model describes three alternative ways of creating unstable stratification: a layer heating from below, a volumetric heating of a fluid with internal heat sources and combination of both factors. The analysis of the model equations shows that under conditions of high intensity of the small-scale convection and low level of heat loss through the horizontal layer boundaries a long wave instability may arise. The condition for the existence of an instability and criterion identifying the threshold of its initiation have been determined. The principle of action of the discovered instability mechanism has been described. Theoretical predictions have been verified by a series of experiments on a laboratory model. The horizontal dimensions of the experimentally-obtained long-lived vortices are 4÷6 times larger than the thickness of the fluid layer. This work presents a description of the laboratory setup and experimental procedure. From the geophysical viewpoint the examined mechanism of the long wave instability is supposed to be adequate to allow a description of the initial step in the evolution of such large-scale vortices as tropical cyclones - a transition from the small-scale cumulus clouds to the state of the atmosphere involving cloud clusters (the stage of initial tropical perturbation).

1 Introduction

The large-scale motions induced by nonuniform heating of the medium exist in the convective zones of the Sun and the stars, in the atmosphere of planets and in the Ocean. The best illustration of such structures is the large-scale, long-lived helical vortices of abnormal intensity, arising in the tropical atmosphere of the Earth and known as tropical cyclones (TC). The key aspect to understanding the TC phenomenon is searching for the mechanisms which control transformation of the permanent atmospheric convective cells with the scales and life time of the cumulus clouds (measuring several kilometers and several hours) to the vortices with the specific structure and characteristic dimension of developed TC (to about 1000 kilometers and more) and the life time of about several days.

The motivation of this work is to study one of the possible initiating mechanisms for the large-scale vortices of TC type and to analyze factors responsible for their generation and evolution using the theoretical model developed in this work and the model-based laboratory experiments.

In the mathematical and laboratory modelling an attempt has been made to identify the most essential effects and processes inherent in the atmosphere of the tropical zone. They can be stated as follows:

1. Two factors responsible for the unstable stratification of the lower atmosphere layer are:
   (a) absorption of the short wave solar radiation by the land and ocean surfaces producing in the atmospheric layer the effect of heating from below;
   (b) volumetric heat release in the atmosphere layer due to condensation and crystallization of the water evaporating from the ocean surface, which occupies a great deal of the tropical area.
2. Practically uniform heating of the atmosphere by solar radiation within $\pm 20^{\circ}$ of the equatorial zone leads to quite insignificant longitude and latitude temperature variations (a monthly average temper-

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ature drop between the equator and these latitudes is only $1 \div 2$ K).

3. Concurrent rotation of the Earth and the atmosphere layer gives rise to Coriolis force.

4. Existence of the large-scale motions in the tropical atmosphere due to a weak latitude gradient (trade winds) and difference in the land and ocean temperatures (monsoons).

5. Development of a deep small-scale turbulent convection in the form of cumulus cloud columns at the background of the interacting large-scale motions. These cloud columns have inessential planar vorticity, their horizontal and vertical scales are of the order of the troposphere height ($10 \div 15$ km) and the life time of each cloud is about 6 hours.

6. Appearance of the cloud clusters differing in size (to about several thousands of kilometers in horisontal directions) and reducing heat transfer of the atmosphere layer at the expense of locking the radiation losses.

7. Formation of the large-scale atmospheric vortices within the cloud clusters.

In contrast to the processes occurring at the middle and high latitudes the large-scale motions in the tropical atmosphere are distinguished by a higher level of stochasticity, which largely depends on the magnitude of the latitude temperature gradient. At the middle latitudes, where the temperature gradient is greater the primary phenomenon is the large-scale circulation of the atmosphere, whereas stochasticity of the large-scale motions is merely the result of their instability. In the tropical zone the situation is quite different. In the tropical atmosphere, especially in the near-equatorial zone, there exist insignificant horizontal temperature nonuniformity and essential instability of vertical stratification. Under these conditions an intensive small-scale convection arises, which penetrates the whole troposphere. In this case stochasticity of the large-scale motions is caused not only by their own instability. The other reason is the evolution of small-scale convective motions with a marked tendency to amalgamation which is realized through the formation of cloud clusters of different size. Therefore, when modelling the tropical atmospheric processes it is essential to take into account the capabilities of the small-scale convection to initiate the large-scale motions.

In order to investigate the conditions and mechanisms of TC initiation it has been suggested to model a tropical atmosphere by a fluid layer with unstable vertical stratification and uniform rotation about its vertical axis. The layer is assumed to meet highly uniform, statistical average conditions of heat and mass transfer in the horizontal directions. The other assumption adopted for the model is a deep turbulent convection with the characteristic scale of the order of the layer thickness.

The initial equations and boundary conditions for a mathematical model have been chosen based on the analysis of the literature, with the emphasis placed on the factors responsible for generation of the large-scale motions.

The classical Rayleigh-Benard problem of convection in a horizontal fluid layer heated from below gave an impetus to the theory of convective stability. At present the horizontal fluid layer still retains its attractiveness as the subject of studies on convective motions, because it can be easily simulated in laboratory conditions and conveniently used for taking different measurements.

Initiation and stability of convection in a plane infinite horizontal layer have been discussed in detail in the monograph (Gershuni, Zhukhovitsky, 1976) for various combinations of the dynamic and thermal conditions. It has been shown that heating from below of the layer with a high thermal conductivity at the boundaries initiates convective motions with a horizontal scale of the order of the layer thickness.

The works (Gershuni, Zhukhovitsky, 1976; Busse, Riahi, 1980; Proctor, 1981) specify the conditions and the work (Gershuni, Zhukhovitsky, 1990) outlines the situations, in which the horizontal dimension of the cell increases. This effect involving a decrease in the critical Rayleigh number is found to be the result of taking into account the finite thermal conductivity of the layer boundary. In the limiting case of the insulated boundaries the horizontal structure dimension tends to infinity (the critical wave number becomes zero). It is worth noting that there is one more factor capable of increasing the scale of convective motions, that is a volumetric heating of the fluid layer by internal sources (Sparrow et.al., 1964; Tritton, Zarraga, 1967; Roberts, 1967).

2 Mathematical model of large-scale convection

2.1 Statement of the problem

A mathematical formulation of the problem is developed in such a way as to provide the most comprehensive description of the basic physical factors operating in the tropical atmosphere and to allow for conditions, which according to the findings of fundamental studies into thermal convection may favour formation of the large-scale structures.

Consider an infinite horizontal plane layer of incompressible fluid, $2h$ thick and rotating about the vertical z-axis with the angular velocity Ω. The problem is described by the equations of natural convection in the Boussinesq approximation for an uniformly rotating fluid (Gershuni, Zhukhovitsky, 1976):

$$v_t^i + (v^i v^j)_j = -\frac{p_i}{\rho} - \epsilon^{ijk} f^j v^k - g^i \beta T + \nu v_{jj}^i$$

$$\begin{aligned} v^j_j &= 0 \\ T_t + (v^j T)_j &= \chi T_{jj} + C_1 \end{aligned} \quad (1)$$

Here v^i are the velocity components, p is the pressure, T is the temperature, ρ is the density; β, ν, χ are the coefficients of the volume expansion, kinematic viscosity and temperature conductivity; $f = 2\Omega$ is twice the angular velocity of the layer rotation, g is the acceleration of gravity, e^{ijk} is the Levi-Chivita symbol. The upper indices denote the vector and tensor components, and the lower indices refer to differentiation with respect to corresponding variables. The tensor indices i, j, k assume values of x, y, z.

The boundary conditions for the velocity and temperature are

$$\begin{aligned} z = 0 \quad &: \quad v^i = 0, \quad T_z = C_2 \\ z = 2h \quad &: \quad v^x_z = v^y_z = v^z = 0, \quad -\kappa T_z = \alpha(T - T_\infty) \end{aligned} \quad (2)$$

where κ is the heat conductivity coefficient of the fluid, α is the heat transfer coefficient of the layer, $(T - T_\infty)$ is the characteristic temperature difference between the fluid and environment.

Here we need to consider the thermal boundary conditions in more detail, since, as already mentioned, they might be one of the factors capable of increasing the horizontal scale of the convective cells.

A boundary condition for the heat transfer at the lower surface can be specified either as the absence of the heat flux or the existence of constant heat flux at the boundary. Assuming that heat transfer at the upper free surface is governed by the Fourier law we specify a more general temperature condition, which describes the heat transfer at the fluid surface to the environment and suggests the existence of heat balance at the interface. The heat transfer coefficient α involved in this condition is a dimensional quantity. The Biot number $Bi = 2h\alpha/\kappa$ is commonly used as a non-dimensional quantity to characterize the heat transfer. When applied at the upper surface such a boundary condition allows us to transport the heat flux out of the fluid to the environment by varying the heat transfer coefficient α. In our experimental studies the heat transfer coefficient is varied by changing the thickness of the transformer oil layer spread over the fluid surface.

The quantities C_1 and C_2 involved in the equations and boundary conditions may assume the following values:

1. $C_1 = q/(\rho c)$, $C_2 = 0$ - for a layer heated with internal sources; here q is the quantity of heat generated by the internal heat sources per unit volume/time, c is the specific heat of the liquid and C_1 is the power of the internal heat sources.

2. $C_1 = 0$ and $C_2 = const$ - for a layer heated from below; C_2 implies the existence of a constant heat flux at the lower boundary.

3. $C_1 = q/(\rho c)$, $C_2 = const$ - for a layer heated from below with internal heat release.

The results of our investigations discussed in this paper refer to two particular situations:

1. The layer is heated by the internal heat sources uniformly distributed in the bulk of the fluid. Its lower boundary is assumed rigid and thermally insulated and the upper boundary is free and transfers heat to the environment.

2. The layer is heated from below. Its lower bounding surface is rigid and thermally insulated, and the upper surface is free to provide the heat flux to the environment. In the laboratory-level experiments such conditions can be effectively simulated by an uniform Joule heat release in a thin plate of metal foil overlying heat-nonconducting massive (foam plastic) which prevents a rapid dispersion of temperature perturbations arising in the fluid.

A combined situation providing two factors responsible for the stratification instability seems to be more suitable for modelling the tropical atmosphere. However, from the previous works (Sparrow et.al., 1964; Gershuni, Zhukhovitsky, 1976) analyzing the conditions for the initiation of convection it follows that in contrast to the case of exclusively internal heating this situation may result in a decreased scale of supercritical motions as the power of internal heat sources increases. Therefore, the primary problem with this situation is to select an optimal ratio between these two layer heating factors, which requires a more thorough analysis of the situation under consideration. This case will be given an independent treatment in our next work.

2.2 Equations for the large-scale fields

The equations of turbulent convection are derived from the initial system 1 by the method of moments (Zimin et.al., 1989; Zimin et.al., 1991a). This approach, developed for flows in a thin fluid layer, enables one to separate motions with low spatial frequencies out of the general structure of turbulent physical fields and to change from three-dimensional to two-dimensional equations. The large-scale components can be separated by expanding in a series the fields of velocity, temperature and pressure

$$\begin{aligned} v^i(x,y,z,t) &= V^i(x,y,t) + (mz-1)a^i(x,y,t) \\ &\quad + \ldots + u^i(x,y,z,t) \\ T(x,y,z,t) &= \Theta(x,y,t) + (mz-1)\vartheta(x,y,t) \\ &\quad + \ldots + T'(x,y,z,t) \\ p(x,y,z,t)/\rho &= P(x,y,t) + (mz-1)d(x,y,t) \\ &\quad + \ldots + p'(x,y,z,t) \end{aligned} \quad (3)$$

Here $m = h^{-1}$; the quantities V^i, Θ, P and a^i, ϑ, d are the transverse spatial moments of zeroth and first order for the corresponding physical fields defined by the following expressions:

Zeroth moment

$$[V^i, \Theta, P(x,y,t)] = \frac{1}{2}m\int_0^{2h}\left[v^i, T, \rho^{-1}p(x,y,z,t)\right]dz$$

First moment

$$[a^i, \vartheta, d(x,y,t)] = \frac{3}{2}m\int_0^{2h}\left[v^i, T, \rho^{-1}p(x,y,z,t)\right](mz-1)\,dz$$

The variables u^i, T', p' have the meaning of turbulent pulsations. In contrast to the universally adopted definition, the pulsations are counted off not from the statistical average values but from the instantaneous large-scale fields defined by the totality of lower moments. These quantities depend on all of the three space variables and allow fulfilment of the boundary conditions.

The number of lower moments to be introduced in the expansion 3 is dictated by the degree of accuracy required to describe the transverse structure of the fields. Thus, the zeroth moments are quite sufficient for treating the large-scale planar flows in a plane layer. To model the large-scale advective motions generated in the atmosphere by a horizontal nonuniformity of the temperature in the form of shear counter-flows involving air injection to the hot spot from the bottom and its withdrawal at the top we have to consider the first order moments in expansion 3. For the problem under consideration there is no need to essentially increase the number of the moments because the structures described by higher moments have small dimensions compared to the layer thickness. The assumption that the model can allow for only two lower moments to describe the temperature fields has been supported by the experimental investigations (Sect. 4.3) which provide the averaged vertical temperature profiles for a plane layer with insulated boundaries heated by internal heat sources. Therefore the mathematical model developed incorporates only zeroth and first moments which seem to be sufficient to describe the mechanism of generating the large-scale structures.

The equations for the n-th order moments can be derived by multiplying each of the equations from system 1 by $(mz-1)^n$ followed by integration over the layer thickness from 0 to 2h. The equations for pulsating quantities are obtained by substituting expansion 3 in the initial system 1 allowing for the equations for the n-th order moments. Thus, the stated problem is reduced to the system of evolutionary equations for transverse space moments corresponding to the physical fields. This system of equations for the zeroth and first order moments is nonclosed because it involves the moments of second order, as well as the zeroth and first order moments of the coupled products of turbulent pulsations. To close the system of equations a number of assumptions have been adopted, which simplify a mathematical formulation of the problem and make it possible to derive semi-empirical expressions to relate the moments of pulsation to those of the large-scale fields. First, introduce a shallow water approximation justified by the fact that the examined structures have horizontal dimensions L which are well above the layer thickness, $L \gg 2h$. Then assume that the parameters of the small-scale turbulent convection in a fluid layer are defined mainly by a vertical heat flux and the effect of the large-scale slowly varying fields on the parameters of the small-scale turbulence can be described by the linear functions of local characteristics of the large-scale fields. Then the closure relations can be derived (Zimin et.al., 1989; Zimin et.al., 1991a) by using a simplified model for a small-scale convection, for example, a model of upgoing thermals applicable for the Rayleigh numbers ranging from 10^6 to 10^{10} (Davenport, Jadson King, 1975; Zimin, Frick, 1988).

We exclude from the equations the zeroth and first order moments of pressure and introduce the stream function $\Psi(x,y,t)$ for the divergence-free field of the zeroth velocity moments, the stream function $\psi(x,y,t)$ and the potential function $\varphi(x,y,t)$ for the field of the first order moments:

$$V^x = -\Psi_y,\ V^y = \Psi_x,\ a^x = -\psi_y + \varphi_x,\ a^y = \psi_x + \varphi_y$$

With the notation:

$$W = V^y_x - V^x_y, \quad w = a^y_x - a^x_y, \quad \gamma = a^x_x + a^y_y$$

(where W and w are z-components of the curl for zeroth and first order velocity moments, respectively, γ is the divergence of the field of first order velocity moments), using the Poisson brackets $\{A,B\} = A_x B_y - A_y B_x$, and the gradient and Laplace operators we arrive at the system of equations for the two lowest moments of the hydrodynamic fields:

$$W_t + \{\Psi, W\} + \frac{1}{3}[\{\psi, w\} - \{\varphi,\gamma\} + \nabla\psi\nabla\gamma + \nabla\varphi\nabla w + 2w\gamma] = \mu\Delta W - \sigma W$$

$$\Theta_t + \{\Psi,\Theta\} + \frac{1}{3}[\{\psi,\vartheta\} + \nabla\varphi\nabla\vartheta + \gamma\vartheta] = C_1 + \mu\Delta\Theta + \delta\gamma - \alpha_1\Theta$$

$$w_t + \{\Psi, w\} + \{\psi, W\} + \nabla W\nabla\varphi + (W+f)\gamma = -\frac{\mu}{4h^2}w - \sigma w$$

$$\gamma_t + \{\Psi,\gamma\} + 2\{\Psi_x, (-\psi_y + \varphi_x)\} + 2\{\Psi_y, (\psi_x + \varphi_y)\} - fw + g\beta h\Delta\Theta = = -\frac{\mu}{4h^2}\gamma - \sigma\gamma$$

$$\vartheta_t + \{\Psi,\vartheta\} + \{\psi,\Theta\} + \nabla\varphi\nabla\Theta = -\frac{\mu}{4h^2}\vartheta$$

$$W = \triangle\Psi, \quad w = \triangle\psi, \quad \gamma = \triangle\varphi \tag{4}$$

The equations of system 4 depend only on the two spatial co-ordinates and the time and describe the behaviour of physical variables in the infinite horizontal plane. The boundary conditions 2 at the horizontal layer surfaces are used when integrating the initial system 1 across the layer thickness and are taken into account when writing the closure relations.

All closure relations enter the right-hand sides of the obtained equations. In the equations for the first order moments these relations are derived only for a turbulent heat and impulse exchange between counter-flows, because under the assumed approximation that $L \gg 2h$ it essentially exceeds the turbulent transfer in the layer plane. The coefficients of turbulent viscosity and thermal conductivity are set equal and denoted by μ. The terms $\mu\triangle W$ and $\mu\triangle\Theta$ describe a turbulent diffusion of the vortex W and temperature Θ averaged over the layer height. This process leads to levelling the spatial inhomogeneities of the large-scale fields and, eventually, to their damping. The terms $\frac{\mu}{4h^2}w$, $\frac{\mu}{4h^2}\gamma$ and $\frac{\mu}{4h^2}\vartheta$ describe the attenuation of the large-scale shear flows w, γ and perturbations of the vertical temperature gradient ϑ at the expense of the turbulent exchange between the lower and upper parts of the layer. The terms σW, σw, $\sigma\gamma$ specify friction at the lower boundary. The term $\alpha_1\Theta$ in the equation for the zeroth temperature moment Θ characterizes heat transfer at the upper surface. The quantity α_1 is related to the heat transfer coefficient of the layer α by $\alpha_1 = \mu Bi/(4h^2)$.

The empirical constants μ, σ, δ must be determined from the experimental studies, which we intend to develop further as the laboratory experiment is updated with the required instrumentation. Nevertheless, based on the model of upgoing thermals (Davenport, Jadson King, 1975) and using the experimental data on the turbulent convection (Kutateladze, Berdnikov, 1984; Zimin, Frick, 1988) and the results of our own experiments we have derived some quantitative theoretical predictions for the desired quantities:

$$\mu \sim 10^{-1}\sqrt{\nu\chi}Ra^{1/3}, \qquad \delta \sim 2\times10^{-3}\frac{\sqrt{\nu\chi^3}}{g\beta h^3}Ra^{5/6}$$

$$\sigma = \frac{\mu}{8h^2}, \qquad \alpha_1 = \frac{\mu}{4h^2}Bi \tag{5}$$

Here, Ra is the Rayleigh number, which depends on the way the layer heating and is defined by the power of internal sources of the magnitude C_1 as

$$Ra = \frac{g\beta q(2h)^5}{\kappa\nu\chi} = \frac{C_1 g\beta(2h)^5}{\nu\chi^2}$$

in the case of volumetric heat release and by a vertical temperature gradient of given magnitude $C_2 = \triangle T/(2h)$ as

$$Ra = \frac{g\beta\triangle T(2h)^3}{\nu\chi} = \frac{C_2 g\beta(2h)^4}{\nu\chi}$$

in the case of layer heating from below. Under different heating conditions the constant numerical coefficients involved in the expressions for empirical constants differ to about 20÷30%. But because of the fact that the procedure used to define empirical constants could by no means yield highly accurate estimations, in both cases the coefficients are conveniently assumed to take values as given in expressions 5.

The analysis of the terms involved in the equations of the mathematical model 4 and the corresponding physical factors shows that the only chance to obtain any new physical effect is related solely to the term $\delta\gamma$ in the equation for zeroth temperature moment Θ. To answer the question of whether the appearance of the term $\delta\gamma$ in the equation of turbulent convection provides conditions for the onset of the long wave instability we shall analyze the stability of the solutions to system 4 against small perturbations.

2.3 Condition for the onset of long wave instability

The system of equations 4 has the solution $\Psi = \psi = \varphi = \vartheta = 0$, $\Theta = C_1/\alpha_1$ corresponding to a turbulent convection in a fluid free of large-scale structures.

Let us examine the stability of such a state against the small nonstationary perturbations varying in time by the exponential law and periodic in the plane (x, y):

$$F_i = A_i exp[-\lambda t + i(k_x x + k_y y)]$$

where i=1-5 corresponds to the quantities W, Θ, w, γ, ϑ; λ is the decrement characterizing the evolution of disturbances; k_x and k_y are the components of the wave vector $\vec{k}$ along the x, y axes.

Substituting perturbations into the evolutionary equations of system 4 and using linear stability theory yield the condition, under which the system of uniform linear equations for perturbation amplitudes has a nontrivial solution

$$(-\lambda + \mu k^2 + \sigma)(-\lambda + \frac{\mu}{4h^2})[(-\lambda + \frac{\mu}{4h^2} + \sigma)^2$$
$$(-\lambda + \mu k^2 + \alpha_1) + f^2(-\lambda + \mu k^2 + \alpha_1)$$
$$-g\beta h k^2\delta(-\lambda + \frac{\mu}{4h^2} + \sigma)] = 0$$

From the analysis of the relation derived we may draw the following conclusions. All small disturbances of W and ϑ decay monotonically with time, their behaviour being independent of the parameter δ. The behaviour of disturbances Θ, w and γ depends on δ.

At $\delta = 0$ and any values of the other problem parameters all small perturbations Θ, w, γ are decaying: in a layer free of rotation (f=0) the perturbations decay monotonically; in a rotating layer ($f \neq 0$) the system has damping perturbations of the frequency f.

For $\delta > 0$ we have obtained the condition which may give rise to the onset of instability in the system

$$\delta > \frac{\mu[(\mu + 4h^2\sigma)^2 + 16h^4 f^2]}{4g\beta h^3(\mu + 4h^2\sigma)} \tag{6}$$

and derived an expression for the wave vectors of the neutral perturbations

$$k_*^2 = \frac{\alpha_1[(\mu + 4h^2\sigma)^2 + 16h^4f^2]}{4\delta g\beta h^3(\mu + 4h^2\sigma) - \mu[(\mu + 4h^2\sigma)^2 + 16h^4f^2]} \quad (7)$$

If condition 6 is satisfied the system involves increasing disturbances when the wave vectors obey $k^2 > k_*^2$.

Thus, the analysis carried out demonstrates that δ-effect is really the factor that initiates instability, causing the formation of the large-scale structures.

2.4 Analysis of factors contributing to the development of the long wave instablity

Let us analyze expression 7 determining the threshold of the instability initiation. According to definition 5 the empirical constants μ, σ, α_1 and δ involved in relation 7 depend on the fluid properties via the kinematic viscosity ν and the temperature conductivity χ and on the Rayleigh number. Moreover, the quantity α_1 can vary independently of the Rayleigh value at the expense of the incoming coefficient of heat transfer α characterizing the thermal properties of the boundary. Fully independent of the parameter Ra is the quantity f, specifying conditions of the layer rotation. In order to differentiate the basic physical factors effecting the evolution of the large-scale instability we need to reduce expression 7 to a dimensionless form.

Substitute in expression 7 the values of empirical constants and introduce two additional dimensionless criteria: the Prandtl number, $Pr = \nu/\chi$ and the Reynolds number, $Re = 4h^2f/\nu$ (the layer height $2h$ is taken as an unit length). Then with the preceding notation k_*, for the dimensionless wave number we arrive at the relation for dimensionless quantities

$$k_*^2 \sim \frac{Bi(2\times 10^{-2}Pr^{-1}Ra^{2/3} + Re^2)}{10^{-2}Pr^{-3/2}Ra^{5/6} - 2\times 10^{-2}Pr^{-1}Ra^{2/3} - Re^2} \quad (8)$$

Expression 8 includes four dimensionless similarity criteria - the Rayleigh, Biot, Reynolds and Prandtl numbers, and thus defines the factors responsible for the development of the long wave instablity in the system. These are the intensity of the small-scale turbulent convection, heat transfer at the layer boundaries, rotation and physical properties of the fluid.

The limiting values for the parameters in expression 8 are those satisfying condition

$$10^{-2}Pr^{-3/2}Ra^{5/6} > 2\times 10^{-2}Pr^{-1}Ra^{2/3} + Re^2$$

which establishes boundaries of the large-scale instability described by the equations of mathematical model 4 with the empirical constants as given by formulae 5.

Relation 8 gives the estimation for the minimum Rayleigh number $Ra_{min} \sim 2^6 \times Pr^3$, above which a long wave instability may occur in the system. This allows us to conclude that the action of the δ-effect is possible only at highly intensive small-scale convection.

The layer rotation produces a stabilizing effect; moreover, in a rotating layer for any $Ra > Ra_{min}$ there exists the limiting maximum value of the Reynolds number above which generation of the large-scale structures becomes unfeasiblle.

For the mathematical model used here it is essential to determine the effect of the boundary thermal properties on the evolution of the large-scale instability.

In both examined cases the lower layer surface is insulated and the free surface transfers heat to the environment according to the Fourier law. Note, however, that the heat transfer at the upper boundary is considered as a factor playing a solo part so that it can be varied independently of the Rayleigh number.

To proceed with the analysis of relation 8 let us fix the Reynolds number and increase the Rayleigh number. Then, in the limit of the Rayleigh number tending to infinity we have the following estimation for the wave number

$$k_*^2 \sim Bi\, Pr^{1/2} Ra^{-1/6} \quad (9)$$

This expression demonstrates that under specified rotation conditions and at given material properties, $Re = const$ and $Pr = const$ an increase in the scale of arising structures (i.e. a decrease of k_*) occurs at decreased heat transfer of the upper boundary (a decrease of the Bi number) and enhanced intensity of the small-scale turbulent convection (a growth of the Ra number).

Having analyzed mathematically eqs. 4 and demonstrated the capabilities of the proposed model to describe the process of generating the large-scale motions we shall attempt in the next Section a physical interpretation of the established effect.

3 Mechanism of anomalous heat transfer

A qualitative explanation for the effect of large-scale structure generation can be gained from the experimentally justified estimation of the processes occurring in a fluid in fully developed turbulent convection. The clearest physical interpretation of the effect is provided by the problem of the layer heated from below.

A large body of laboratory-scale investigations on turbulent convection in a horizontal layer (Davenport, Jadson King, 1975; Kutateladze, Berdnikov, 1984; Zimin, Frick, 1988) shows that at Rayleigh numbers $Ra = 10^6 \div 10^{10}$, near the horizontal surfaces, there exist thin boundary layers, several millimeters thick with unstable temperature stratification. At the external boundaries of these layers one can observe the generation of thermals. They are distinguished as "cold" thermals - at the upper boundary and "hot" thermals - at the lower boundary. The external view of the thermals is given in Fig. 1 as provided in (Album of Fluid Motion, 1982). After separating from the boundary layers the thermals

Fig. 1. Thermals (columns in the form of mushrooms) rising from the surface of the heated copper plate (the figure is borrowed from "An Album of Fluid Motion").

move with accelerating motion until they reach the opposite boundary. As thermals move they involve in their motion the surrounding fluid.

The neighbouring thermals through capturing the interspace fluid reduce the fluid pressure and are attracted together to spontaneously form the groups of jointly rising thermals. As the result, the scale of convective motions increases from the size of a separate thermal to that of a group comparable to the layer height.

A further growth of the motion scale in the horizontal direction is possible with a random small temperature perturbation in the layer in the form of a hot spot characterized by a temperature difference $(T_1 - T_2)$ between its center and periphery. Such a temperature perturbation generates a shear flow, which involves the upgoing flow at the spot center and downgoing flow at the periphery. The motions in the layer are schematically represented in Fig. 2. For example, the cold thermal separated from the upper boundary layer is trapped by the shear flow $v^x = a^x(mz - 1)$ directed in the upper part of the layer $(h < z < 2h)$ toward the spot periphery and deflected by it for a distance of l_1. The bottom flow $(0 < z < h)$ deflects the thermal in the opposite direction for a distance of l_2. Since at the accelerated downward motion the cold thermals reside in the upper part of the layer for a longer time, than in the lower part $l_1 > l_2$ and the resulting drift of the cold thermals is in the direction of the spot periphery. Similar reasoning shows that hot thermals from the lower boundary layer move toward the spot center.

Thus, in the region of the hot spot there is a layer-average convective heat flux in the direction of the horizontal temperature gradient, i.e a heat transfer takes place out of the colder to warmer regions, which increases the initial temperature difference $(T_1 - T_2)$. This, in turn, suggests that the heated fluid elements rising in the center of the hot spot now possess a greater heat reserve per unit mass Q_i. On reaching the upper boundary the warm fluid moves along it toward the spot periphery under the action of the shear flow. While doing this it transfers heat to the environment and becomes colder. Moreover, the greater is the heat reserve of the floated fluid the longer time it takes to transfer the heat and, consequently, the longer distance along the upper boundary must be covered before reaching the temperature, at which the fluid begins to sink to the bottom. This means that with increasing intensity the horizontal scale of the initial heat perturbation also increases.

The explanation offered for the mechanism of "anomalous" heat transfer also holds true in the case of internal layer heating, with the difference that in this situation there is only a "cold" boundary layer with thermals close to the upper boundary and the upgoing warm fluid elements are generated through the whole fluid volume outside the boundary layer region.

It should be emphasized that the established mechanism of the long wave instability - that of "anomalous" heat transfer - can take place only in conditions of low heat transfer at layer boundaries. Otherwise, the arising large-scale temperature perturbations and the resulting convective motions decay rapidly due to a heat loss through the horizontal boundaries.

Of particular importance is also the fact that the above mechanism operates both in the rotating layer and in the layer free of rotation.

4 Modelling in laboratory conditions

4.1 Experimental setup and test conditions

Laboratory experiments and their results have been for the first time presented in (Zimin et.al., 1991b). However, in those days the theoretical model had yet to be analyzed and therefore, the experiments were made based only on some preliminary hypotheses. Among them, the hypothesis of necessity to combine the high intensity of the small-scale turbulent convection with the low level of heat losses through the horizontal boundaries was of primary interest. At present its importance is supported by the theoretical analysis of the large-scale convection equations.

Realization of the Rayleigh convection in laboratory conditions has been the problem because the above approaches are contradictory. In the experiments described below this contradiction has been overcome at the expense of Joule heating in an electrolyte, thermally insulated at its bottom, and of creation of the intensive

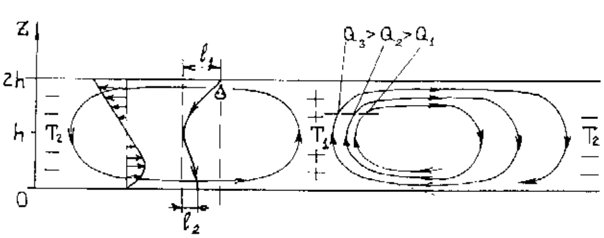

Fig. 2. Flow diagram demonstrating the principle of the "anomalous" heat transfer mechanism.

heat flux to the upper free boundary by the high temperature difference between the fluid and the environment at low heat-transfer coefficient. In such a way it appears possible to induce an intensive small-scale convection and secure good heat insulation of the fluid layer horizontal boundaries. In this case the heat perturbations, occurring in the layer, as well as the perturbation rates generated by them, are damped slowly.

In the experiments a foam plastic cuvette, filled with electrolyte, with inner size 0.7×0.7 m^2 in plane and height 0.06 m, was mounted on a horizontal platform able to rotate around the vertical axis. The fluid layer was uniformly heated by passing the alternating current through it. The heat removal from the fluid was due to evaporation and natural convection of air over the fluid free surface. The upper surface of the cuvette was open. Variation of the heat-transfer coefficient was achieved by applying a continuous thin layer of transformer oil to the electrolyte surface which kept the water from evaporating. The heat-transfer coefficient was determined experimentally by dependence of the heat quantity, dispersed in the environment from the upper electrolyte surface per unit time in a stationary regime, on the temperature drop between the upper electrolyte surface and the environment. The fluid motion was visualized with the aid of aluminium powder which allowed us to observe the upper electrolyte surface; the structures occurred were registered by a videocamera or an ordinary camera mounted on the rotating platform.

Let us analyze the experimental results taking into account predictions and estimations concerning the role of different physical factors in generating the large-scale structures.

4.2 Classification of the motion regimes

In laboratory experiments we studied motion regimes realized by changing three dimensionless similarity criteria, namely, the Rayleigh, Biot and Reynolds numbers. No consideration was given to the dependence of regimes on the Prandtl number.

Tests were made to study four combinations of the rotation and thermal insulation regimes: at $Re = 0$ (no rotation) and in the range $Re{\sim}200{\div}400$ (rotating layer), and two variants of the heat-transfer on the upper electrolyte surface at $Bi{\sim}1.3{\div}2.0$ (the surface was not insulated) and at $Bi{\sim}0.35{\div}0.45$ (the surface was insulated with a thin film of transformer oil).

In a series of experiments, carried out in the absence of rotation ($Re = 0$), soon after destabilizing the mechanical equilibrium the convective motions occured in the electrolyte layer in the form of rolls. Their transverse dimension is $1.0{\div}1.5$ H and their longitudinal dimension is 4 H (here H denotes the fluid layer height). Then as Ra increases, the structure of cells changes crucially, i.e. the rolls change to polygonal cells with characteristic horizontal scale $L{\sim}1{\div}2$ H. The large-scale cells with $L{\sim}4$ H appeared at the background of the small-scale ones at $Ra_*{\sim}9.3\times10^6$, $Bi{\sim}1.5$.

The improvement in the thermal insulation at the upper surface of the electrolyte by spreading the oil layer reduced the Biot number by a factor $3.0{\div}4.5$ and caused the large cells to increase up to $L{\sim}6$ H. With increasing Rayleigh number, changes in configuration of the convective structures went in the same order as in the experiments without applying the oil. The existence of large-scale cells was seen at $Ra_*{\sim}6.2\times10^6$, $Bi{\sim}0.42$.

A comparison of these experimental results with the linear theory gave rather interesting and even some unexpected results (taking into account the way of defining the empirical constants in closure relations). According to expression 8 in the absence of rotation ($Re = 0$) the critical wave numbers k_* were calculated for the values of numbers Ra and Bi given above. Theoretical estimations of the sizes of convective structures proved to be excessive by only a factor $2.0{\div}2.5$ as compared to the structure scales examined.

From theoretical and experimental results in the case $Re = 0$ it follows that the fluid layer rotation is not a necessary condition for performance of the long wave instability mechanism. The mechanism of "anomalous" heat transfer provides an increase in the convective structure scale even when the rotation is absent.

However, if our purpose is to model the origination of large-scale TC type vortex with spiral structure, then the layer rotation resulting in the Coriolis force is of crucial importance as it leads to generation of the vortex motions in the horizontal plane and transforms every convective cell into a spiral vortex.

Experiments showed that the most interesting regimes were those with relatively low values $Re{\sim}200{\div}330$, in which the large-scale cyclonic vortices with $L{\sim}4{\div}6$ H were observed. In the absence of the oil film, i.e. at a higher rate of heat losses, the large-scale vortices with $L{\sim}4$ H arise at the background of small-scale vortices at $Ra_*{\sim}9.3\times10^6$, $Re{\sim}280$, $Bi{\sim}1.5$. These large-scale vortices were penetrated by upgoing and downgoing flows of a small-scale circulation and do not have a clearly defined shape. The life time of such vortices was $5{\div}10$ s. The improvement in the thermal insulation at the upper surface of the electrolyte through spreading the oil decreases the Biot number by a factor of $3{\div}4.5$ thus increasing the life time of the large-scale formations by an order of magnitude. In this case the large-scale ($L{\sim}6$ H), long-lived vortices were observed at $Ra_*{\sim}6.2\times10^6$, $Re{\sim}320$, $Bi{\sim}0.40$. Such large-scale vortices greatly suppress the small-scale convection. Viewed from the top, they are seen as the spirals unwinding from the center (Fig. 3), where the straight dark lines correspond to the downgoing cold fluid.

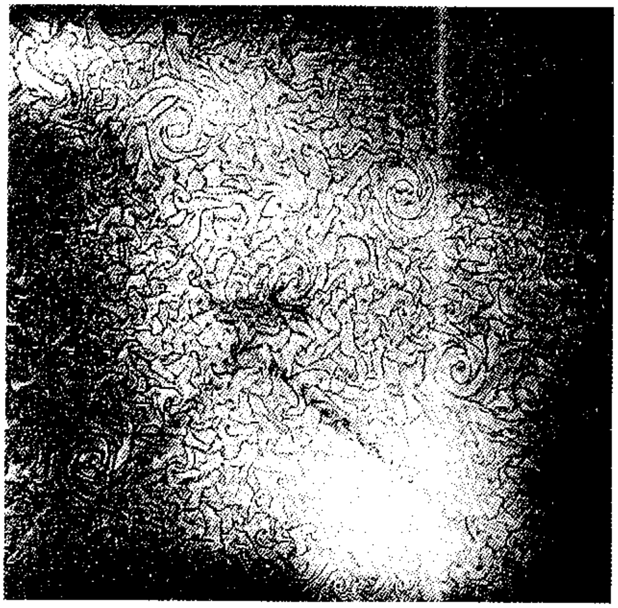

Fig. 3. Large-scale vortices at the background of the small-scale convection at $Ra\sim1.0\times10^7$, $Re\sim380$, $Bi\sim0.41$.

4.3 Temperature profiles and Fourier spectra of temperature fields

With our special automated system for accumulating and treating data on temperature fields we analyzed in each experiment the evolution of the average temperature profiles across the layer depending on the Rayleigh number. An averaging for every point of each profile was made using the results from 2000 measurements performed with the frequency of 25 readings per a second. Fig. 4 presents the profiles for rotating electrolyte layer without the oil at the surface. The study of these relations shows that if the parts of the profiles assigned to the upper boundary layer are ignored, it becomes possible to describe completely the rest part using only zeroth and first moments of the temperature field from expansions 3. This justifies our suggestion for considering in the mathematical model only the two lowest moments of the temperature field.

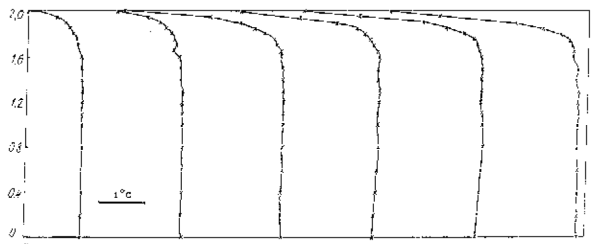

Fig. 4. Evolution of mean vertical temperature profiles in the layer of electrolyte with the surface free of the oil film for $\Omega = 0.21\,s^{-1}$.

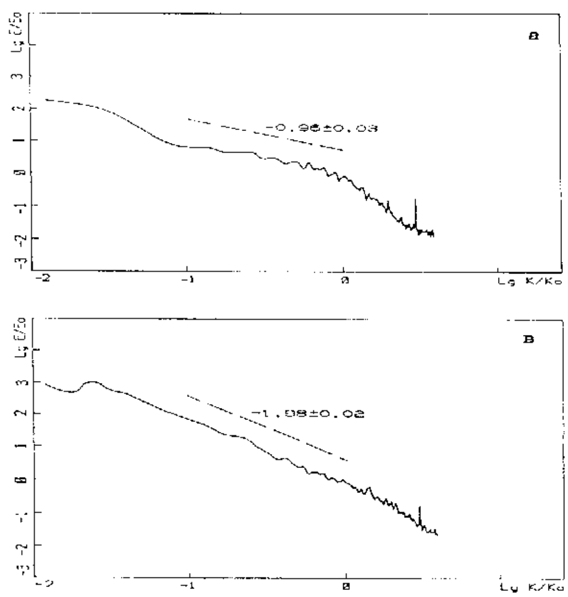

Fig. 5. Fourier spatial spectra of the temperature fields prior (a) and subsequent (b) to the generation of large-scale vortices in the electrolyte layer.

Under laboratory conditions we have also obtained the spatial Fourier spectra of the temperature fields before and after generation of the large-scale vortex structures. Measurements were made within the core of the electrolyte layer (no consideration was given to the upper boundary layer). The signal is taken from the differential copper-constantan thermocouple. One of the thermocouple junctions was placed in a Dewar at room temperature and the other was immersed in the layer of electrolyte circumscribing the circles of different diameter as the cuvette rotates. The centers of these circumferences coincide with the axis of the cuvette rotation. Thus, the thermocouple signal was an one-dimensional space cut of the temperature field. Spectra were calculated from 1024 measurements using the fast Fourier transform.

In Fig. 5 there are the spatial Fourier spectra of the temperature fields before (see top diagrams) and after (see bottom diagrams) generation of the large-scale vortex structures in the electrolyte. These relations, taken from various parts of the fluid layer core, are analogous. This fact is explained by a high uniformity degree of heat/mass transfer conditions in the horizontal directions.

At the numbers $Ra < Ra_*$ there is a break in the spectrum of normalized energy E for the wave numbers k_o, corresponding to the structures with a specified dimension by an order of the electrolyte layer height H. As the unit of measurement E we took the value E_o equal to the energy of motions with the wave number k_o. As the large-scale structures are developed, the distribution of the form $E\sim k^{-2}$ is established over the whole range of wave numbers.

By comparison of the diagrams it is seen that the energy of motions with chacteristic scales on the order 10 H rises on the low spectrum by more than an order of magnitude. This suggests that a long wave instability develops in the system.

Spectra from Fig. 5 were obtained by averaging over six realizations. Indices of power in the energy drop law and an error on their determination were found by the method of least squares for the rectilinear parts of these spectra falling in the range of the wave number from $0.1k_o$ to k_o, which corresponds to a change in the typical size of structures from 10 H to H, respectively.

The authors of the work (Zimin et.al., 1991a) presented similar data which were obtained on processing the results from experiments carried out during an expedition "Typhoon-89" aboard the research ship "Akademik Korolyov." In particular, it was shown that as the tropical depression became deeper, one observed changes in the spatial spectra of radiointensive atmospheric temperatures, calculated by using data from the multichannel SHF-radiometric complex on the board of the ship. On intensification of the tropical depression, the distribution of the form $E{\sim}k^{-2}$ was established.

5 Conclusions

Theoretical analysis of conditions contributing to the formation of large-scale structures has shown that most crucial for this process is the combination of two factors: the high intensity of the small-scale convection and the low level of the heat loss through the region boundaries. At a low heat transfer of the layer boundaries the appearance of the long wave instability might be expected, which is due to the development of random thermal perturbations creating a horizontal temperature gradient. Such an instability is accompanied by initiation of the advective heat flux in the direction of the horizontal gradient of the average temperature which intensifies the original thermal perturbation and increases its horizontal scale.

Realization of the above conditions in the laboratory model allowed us to obtain experimentally the long-lived vortices with the horizontal dimensions 4÷6 times as large as the layer height.

From the viewpoint of geophysical applications the herein discovered mechanism of the large-scale instability allows one to fill the gaps in studying the atmosphere phenomenon of tropical cyclones through the description of the initial step in the evolution of the large-scale vortices of the TC type, i.e. the transition from the small-scale cumulus clouds to a state of the atmosphere with cloud clusters (the stage of initial tropical perturbation).

The results obtained in this work can be applied to study geophysical problems, in particular, to model such processes of large-scale structure generation in the tropical atmosphere as cloud clusters and tropical cyclones.

Acknowledgements. This work was supported by Russian Foundation of Fundamental Investigation under Grant N 95-01-01094a.

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